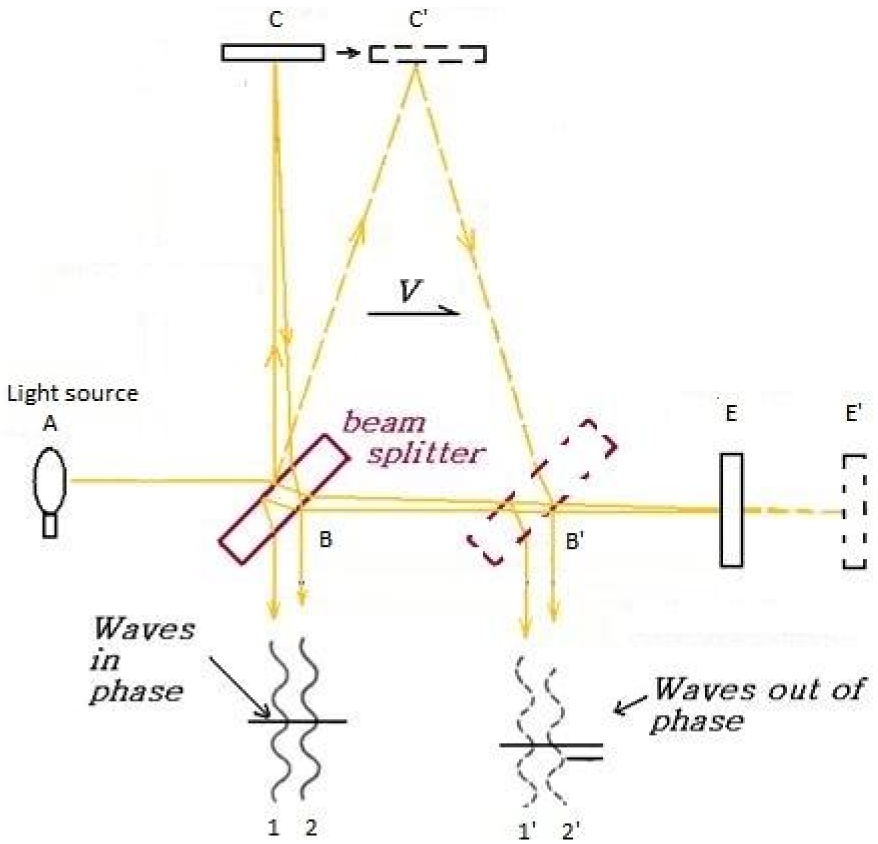
Michelson Interferometer Calculations



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Interference
pattern observed here

Michelson interferometer

1.0 Description

A schematic diagram of the interferometer is shown above. The Interferometer is moving with the earth from left to right at velocity 'v'. Light from a source (A) is reflected through a right angle by a "half silvered" mirror (B) towards a mirror (C). The light also passes through the half silvered mirror towards a mirror E.
The reflected light from mirror C passes through the half silvered mirror and the light reflected from the mirror E is reflected by the half silvered mirror so that the two returning rays 1 and 2 blend together. This generates interference between the waves as illustrated. The dotted positions and B' C' and E' show the new location of the apparatus due to movement of the earth (somewhat exaggerated for clarity) as the light passes along its length and breadth. The interfering waves will also move to positions 1'and 2' as the apparatus moves.

To use the interferometer, it is rotated through 90^0 about mirror B so that the axis BE moves through a right angle to the axis BC. The different time of transit in the two arms can be measured by counting the number of interference fringes which pass across the viewing point. Each fringe represents a movement of one wavelength of light.

The velocity at which the interferometer is moving is denoted by 'v'. The speed of light will denoted by 'c'

The length of the interferometer arms, (both of equal length), from point B to E and from B to C is denoted by L

Calculating the transit time for light to travel from B to E and back

Let the time for light to travel from B to E' be t_1 and the time to return to B' be t_2.

Whilst the light is travelling from B to E' the interferometer will move vt_1 (distance = speed x time)

Light from the source must travel from the half silvered mirror B to the reflector E' and the distance covered will be $L + vt_1$

Since light travels at the speed c, the same distance will also be ct_1

$ct_1 = L + vt_1$ which when rearranged can be expressed as $t_1 = \frac{L}{(c-v)}$ .... (1)

For the return journey the distance covered by the light in going from E' to B', will be less than for the journey from B to E' because B is moving towards E. This distance will be $L - vt_2$.

The time for light to transit from E' to B' can also be expressed as ct_2

Thus $ct_2 = L - vt_2$ or $t_2 = \frac{L}{(c+v)}$ ....(2)

Thus the total time for light to travel from B to E' and back again to B' is:-

$$t_1 + t_2 = \frac{L}{(c-v)} + \frac{L}{(c+v)}$$

If we add these two expressions using a common denominator of (c-v)(c+v) which equals $(c^2 - v^2)$, we have:-

$t_1 + t_2 = \frac{L(c+v) + L(c-v)}{(c^2 - v^2)} = \frac{2Lc}{(c^2 - v^2)}$ Dividing top and bottom of this expression

by c and rearranging, we have:-

$$t_1 + t_2 = \frac{2L/c}{1-v^2/c^2} \quad \text{...(3)}$$

Calculating the transit time for light to travel from B to C', and back to B'

let the time for light to travel from B to C' be t_3, and the time for light to travel back from C' to B' be t_4

As before during the time t_3 the mirror E moves to the right by a distance vt_3 and during the same time, the light travels a distance ct_3. the light will travel on the hypotenuse of a right angled triangle as can be seen from the diagram, and thus we can use Pythagoras' theorem :-

$$(ct_3)^2 = L^2 + (vt_3)^2$$

This can be expanded to $c^2 t_3^2 = L^2 + v^2 t_3^2$

Thus $L^2 = c^2 t_3^2 - v^2 t_3^2 = t_3^2(c^2 - v^2)$

Taking square roots of both sides of the equation and rearranging:-

$$t_3 = \frac{L}{\sqrt{c^2 - v^2}}$$

For the return trip from C' to B' the time of travel will be the same as can be seen from the symmetry of the figure, therefore:-

$$t_4 = \frac{L}{\sqrt{c^2 - v^2}}$$

The total time of transit from B to C and back again, will be:-

$$t_3 + t_4 = 2t_3 = \frac{2L}{\sqrt{c^2 - v^2}}$$

Dividing top and bottom by c and rearranging:-

$$t_3 + t_4 =. \frac{2L/c}{\sqrt{1-v^2/c^2}} \quad \text{..(4)}$$

We are now in a position to compare the transit time of the two beams of light.

The time difference (Δt) in transit times for light to pass between BE and BC is

$(t_3 + t_4) - (t_1 + t_2)$ which from equations (3) and (4) is :-

$$(\Delta t) = \frac{2L/c}{\sqrt{1-v^2/c^2}} - \frac{2L/c}{(1- v^2/c^2)} \quad \text{..(5)}$$

The only unknown in this equation is v (the velocity of earth in space).

This time difference Δt is calculated as follows:-

The number of fringes which pass across the viewing point when the interferometer is swung through a right angle (as described in section 1.0) is counted. let this be 'n'

The time difference Δt is represented by the time taken for light to pass through 'n' wavelengths.

let the wavelength of the light source be λ. c is the speed of light.

Time = distance /speed, thus:- $\Delta t = n\lambda/c$, and from equation (5)

$$n\lambda/c = \frac{2L/c}{\sqrt{1-v^2/c^2}} - \frac{2L/c}{1-v^2/c^2}$$

$$n\lambda = \frac{2L}{\sqrt{1-v^2/c^2}} - \frac{2L}{1-v^2/c^2}$$

v is the only unknown in this equation, and thus can be evaluated. $n\lambda$ is the number of fringes multiplied by the wavelength of light used.

Fitzgerald and Lorenz Contraction

The proposition was that the length of the arm in the direction of travel will change with speed.

(The transverse arm (BC) which is not moving along its length will not change in length).

We will now examine how Length in the direction BE would change with speed if the transit time is to be unchanged.

Length in this direction will be designated L_1.(length in the direction BC will remain at L)

If the transit time is unchanged, then $(t_1 + t_2) = (t_3 + t_4)$. Therefore from equations 3 and 4,

$$\frac{2L_1/c}{(1-v^2/c^2)} = \frac{2L/c}{\sqrt{1-v^2/c^2}}$$

which can be re-arranged as:-

$$L_1 = L\sqrt{1-v^2/c^2}$$

This means that the transit time will be the same in both directions at right angles if:-

Length in the direction of travel reduces by a factor of $\sqrt{1-v^2/c^2}$

Change of the rate of passage of time

Equation (3) shows the time for light to pass along and back on the axis along which the interferometer is moving.

$$t_1 + t_2 = \frac{2L/c}{1 - v^2/c^2}$$

Fitzgerald /Lorenz proposed that Length varies with speed so that the time of transit $t_1 + t_2$ would remain constant with changing speed, which the Michelson Morley experiment had shown.

Light, however always travels at the speed of light, and thus there is an apparent conflict. If length shortens with speed, the time of transit of the light should also shorten as it passes along this reduced distance.

The Fitzgerald /Lorenz proposition was, however, that the time of transit $t_1 + t_2$ would remain constant with changing speed.

The only way to reconcile this conflict is to conclude that the passage of time must slow down, so that the reduced distance can be covered by light in a time which has not been reduced. Time will therefore slow by a factor of

$$\sqrt{1-v^2/c^2}$$

We now know from Einstein's special relativity that this is true.

Dependence of mass on velocity

It is not possible to deduce that mass is affected by velocity directly from the Michelson Morley experiment, but it was shown by Albert Einstein that a related correction factor $\frac{1}{\sqrt{1-v^2/c^2}}$ applies to mass.

The molecules in a gas are in constant motion, and when it is heated their speed increases. (It does not matter in which direction the molecules move) This will therefore result in an increase in mass.

let us take the gas at some known temperature, and designate its mass as M_0.

If we heat the gas, the speed of the molecules will increase by a velocity which we will designate as "u"

We can calculate the mass of the gas (M) at the higher temperature, simply by

multiplying the mass (M_0) by the factor $\frac{1}{\sqrt{1-u^2/c^2}}$.

Thus $M = \dfrac{M_0}{\sqrt{1-u^2/c^2}}$ ....(5)

This led to the calculation of the most famous equation in all physics.

We can expand equation (5) using Maclaurin's expansion. The equations become unwieldy to write, using standard typing notations, and if we let $u^2/c^2 = x$, the expansion gives

$$\frac{1}{\sqrt{1-x}} = 1 + x/2 + 3x^2/8 + 15x^3/48 + 105x^4/384 + \ldots\ldots\ldots\ldots$$

For small values of x the terms become vanishingly small, beyond the second term, and we will not consider any terms beyond this.

Thus replacing x for u^2/c^2 , and limiting the expression to the first two terms is

$$\frac{1}{\sqrt{1-u^2/c^2}} = 1 + \frac{u^2}{2c^2} \quad .$$

Thus from equation (5)

$$M = M_0 \left[1 + \frac{u^2}{2c^2}\right]$$

simplifying,

$$M = M_0 + \frac{M_0 u^2}{2c^2} \quad .$$

Therefore:-

$$M - M_0 = \frac{M_0 u^2}{2c^2}$$

Multiplying this expression by c^2 we have:-

$$Mc^2 - M_0c^2 - 1/2\ M_0u^2$$

The expression $1/2\ M_0u^2$ is the kinetic energy (internal energy) in the molecules resulting from the increase in speed of movement (temperature increase), which we will designate as 'e'

Thus the increase in mass due to heating $(M - M_0)$ which we will designate as 'm' multiplied by the square of the speed of light(c^2) is equal to the increase in internal energy (e)

Thus $e = mc^2$

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